Doctoral School of Information and Biomedical Technologies Polish Academy of Sciences

Subject: Numerical methods in optimal control and shape optimisation of coupled problems with incompressible flows.

Supervisor: Prof. dr hab. inż. Jan Sokołowski, tel.: (+48) 22 38 10 229 (Jan.Sokolowski@ibspan.waw.pl)

Co-supervisor: dr Katarzyna Szulc, tel.: (+48) 22 38 10 215 (Katarzyna.Szulc@ibspan.waw.pl)

Place of research: Systems Research Institute, Polish Academy of Sciences, ul. Newelska 6, 01-447 Warszawa, Poland

Scientific discipline: Technical Computer Science and Telecommunications

Recruitment form: Interview

Available positions: 1

Project description:

Shape optimisation is recently intensively developed technique due to the large range of applications in different fields of sciences, such as engineering industry and life sciences. Among them we can point out the structural mechanics, fluid or gas mechanics as well as optimal control. In very simple words, the shape optimization problem can be formulated as a minimisation of a given cost (shape) functional given for some class of (admissible) domains. This functional depends on the domain directly but also indirectly through the solution to a boundary value problem defined for this domain. The general idea is to find an optimal shape of an object which satisfies given constraints, with respect to one or more costs.
Problems such as interaction of • gas/fluid flow surrounded the structure or • fluid inside a structure are prime examples of coupled models that we are going to consider in this project. Both cases are physically relevant: the case where the structure is ”inside”, immersed in the fluid, as in the case of a submerged submarine or airborne aircraft, and the case where the structure is ”outside” the fluid, as it occurs in a jet engine or in case of blood flow in the arteries.
The systems are governed by coupled models with elliptic partial differential equations. Analysis of those models is carried in two steps: the first step is the mathematical modeling of coupled problems with free boundaries, and the second step is to develop the discrete model that could be used for numerical solutions to optimal control and shape optimization tasks. The proposed subject is interesting from the point of view of practical applications. The main objective is to study the sensitivity of such a model with respect to the continuous deformations of geometrical domains. We want to minimize a given functional due to the changes of the geometry and topology of the domain. We can consider two types of such model. First one is to study the Stokes equation which models the flows of a fluid with high speed. In this case we consider a control of two function parameters: the source and the boundary of geometrical domain. The second one is to study the elasticity problems where the variational inequalities appear. In this case the topology of the domain is perturbed by creating small hollow voids inside. With respect to the above models the following research objectives are put forward and proposed for investigation: 1. Analysis of mathematical model of a structure (obstacle) immersed in an incompressible fluid. 2. Shape optimization of the structure (obstacle) with an eye on minimizing the drag near the interface (free boundary in general). 3. Analysis of optimal control of moving obstacle. 4. Numerical analysis of mathematical models obtained by theoretical studies.

Bibliography:

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